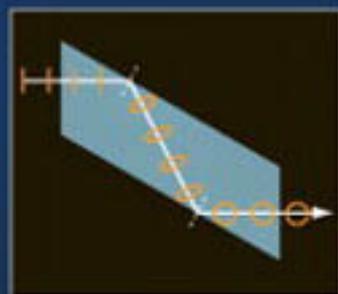
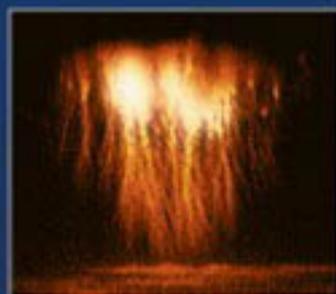
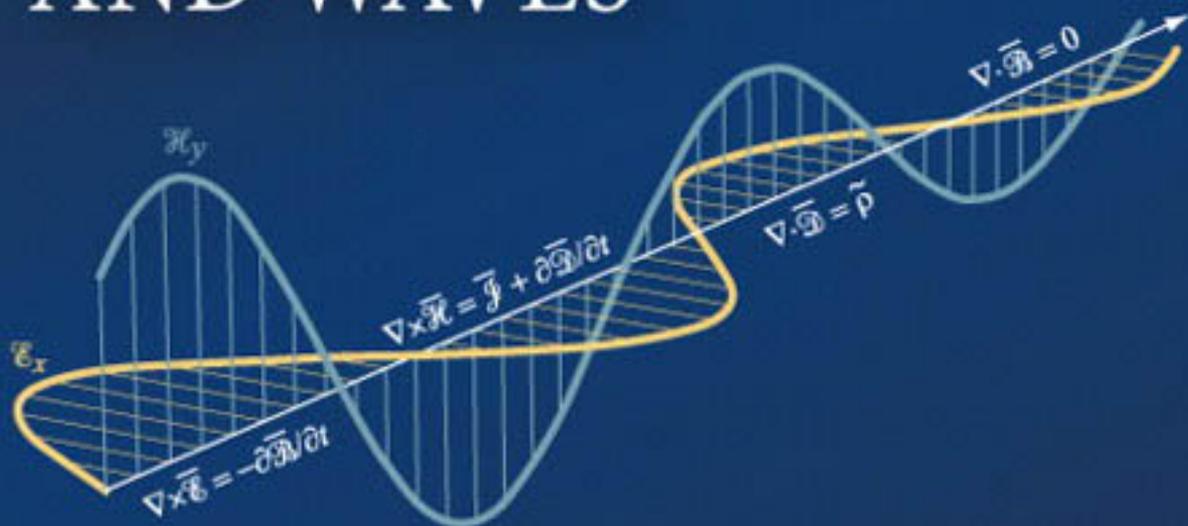


Second Edition

# ENGINEERING ELECTROMAGNETICS AND WAVES



Umran S. INAN

Aziz S. INAN

Ryan K. SAID

# ENGINEERING ELECTROMAGNETICS AND WAVES

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Second Edition

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**Library of Congress Cataloging-in-Publication Data**

Inan, Umran S.

Engineering electromagnetics and waves / Umran S. Inan, Aziz S. Inan, Ryan K. Said.  
pages cm

Includes bibliographical references and index.

ISBN 978-0-13-266274-1 (alk. paper)

1. Electromagnetic theory. I. Inan, Aziz S. II. Said, Ryan K. III. Title.

QC670.I52 2014

530.14'1-dc23

2014002226

10 9 8 7 6 5 4 3 2 1

**PEARSON**

ISBN 10: 0-13-266274-4

ISBN 13: 978-0-13-266274-1

*To our families*

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# Preface

This book provides engineering students with a solid grasp of electromagnetic fundamentals and electromagnetic waves by emphasizing physical understanding and practical applications. The topical organization of the text starts with an initial exposure to transmission lines and transients on high-speed distributed circuits, naturally bridging electrical circuits and electromagnetics.

*Engineering Electromagnetics and Waves* is designed for upper-division (3rd and 4th year) college and university engineering students, for those who wish to learn the subject through self-study, and for practicing engineers who need an up-to-date reference text. The student using this text is assumed to have completed typical lower-division courses in physics and mathematics as well as a first course on electrical engineering circuits.

## Key Features

The key features of this textbook are

- Modern chapter organization, covering transmission lines before the development of fundamental laws
- Emphasis on physical understanding
- Detailed examples, selected application examples, and abundant illustrations
- Numerous end-of-chapter problems, emphasizing selected practical applications
- Historical notes on the great scientific pioneers
- Emphasis on clarity without sacrificing rigor and completeness
- Hundreds of footnotes providing physical insight, leads for further reading, and discussion of subtle and interesting concepts and applications

## Modern Chapter Organization

We use a physical and intuitive approach so that this engineering textbook can be read by students with enthusiasm and interest. We provide continuity with electric circuit theory by first covering transmission lines—an appropriate step, in view of the importance of transmission line concepts, not only in microwave and millimeter-wave applications but also in high-speed digital electronics, microelectronics, integrated circuits, packaging, and interconnect applications. We then cover the fundamental subject material in a logical order, following the historical development of human understanding of electromagnetic phenomena. We base the fundamental laws on experimental observations and on physical grounds, including brief discussions of the precision of the fundamental experiments, so that the physical laws are easily understood and accepted.

Once the complete set of fundamental laws is established, we then discuss their most profound implications: the propagation of electromagnetic waves. We begin this discussion with the propagation of waves through empty space or unbounded simple media. We then discuss the reflection and refraction of electromagnetic waves from simple planar boundaries, followed by the guiding of electromagnetic waves within planar metallic or dielectric structures. We conclude with an introduction to field-matter interactions and electromagnetic wave propagation in metamaterials.

## Emphasis on Physical Understanding

Future engineers and scientists need a clear understanding and a firm grasp of the basic principles so that they can interpret, formulate, and analyze the results of complex practical problems. Engineers and scientists nowadays do not and should not spend time obtaining numerical results by hand. Most of the number crunching and formula manipulations are left to computers and packaged application and design programs, so a solid grasp of fundamentals is now more essential than ever before. In this text we maintain a constant link with established as well as new and emerging applications (so that the reader's interest remains perked up), while at the same time emphasizing fundamental physical insight and solid understanding of basic principles. We strive to empower the reader with more than just a working knowledge of a dry set of vector relations and formulas stated axiomatically. We supplement rigorous analyses with extensive discussions of the experimental bases of the laws, of the microscopic versus macroscopic concepts of electromagnetic fields and their behavior in material media, and of the physical nature of the electromagnetic fields and waves, often from alternative points of view. Description of the electrical and magnetic properties of material media at a sufficiently simple, yet accurate manner at the introductory electromagnetics level has always been a challenge, yet a solid understanding of this subject is now more essential than ever, especially in view of many applications that exploit these properties of materials. To this end we attempt to distill the essentials of physically-based treatments available in physics texts, providing quantitative physical insight into microscopic behavior of materials and the representation of this behavior in terms of macroscopic parameters. Difficult three-dimensional vector differential and integral concepts are discussed when they are encountered—again, with the emphasis being on physical insight.

### **Detailed Examples and Abundant Illustrations**

We present the material in a clear and simple yet precise and accurate manner, with interesting examples illustrating each new concept. Many examples emphasize selected applications of electromagnetics. Over 190 illustrative examples are detailed over eleven chapters, with five of the chapters having at least 20 examples each. Each example is presented with an abbreviated topical title, a clear problem statement, and a detailed solution. In recognition of the importance of visualization in the reader's understanding, especially in view of the three-dimensional nature of electromagnetic fields, over 500 diagrams, graphs, and illustrations appear throughout the book.

### **Numerous End-of-Chapter Problems**

Each chapter is concluded with a variety of homework problems to allow the students to test their understanding of the material covered in the chapter, with a total of over 400 exercise problems spread over eleven chapters. The topical content of each problem is clearly identified in an abbreviated title (e.g., “Digital IC interconnects” or “Inductance of a toroid”). Many problems explore interesting applications, and most chapters include several practical “real-life” problems to motivate students.

### **Historical Notes**

The history of the development of electromagnetics is laden with outstanding examples of pioneering scientists and development of scientific thought. Throughout our text, we maintain a constant link with the pioneering giants and their work, to bring about a better appreciation of the complex physical concepts as well as to keep the reader interested.

### **Emphasis on Clarity without Sacrificing Rigor and Completeness**

This textbook presents the material at a simple enough level to be readable by undergraduate students, but it is also rigorous in providing references and footnotes for in-depth analyses of selected concepts and applications. We provide the students with a taste of rigor and completeness at the level of classical reference texts—combined with a level of physical insight that was so well exemplified in some very old texts—while still maintaining the necessary level of organization and presentation clarity required for a modern textbook. We also provide not just a superficial but a rigorous and in-depth exposure to a diverse range of applications of electromagnetics, in the body of the text, in examples, and in end-of-chapter problems.

### **Hundreds of Footnotes**

In view of its fundamental physical nature and its broad generality, electromagnetics lends itself particularly well to alternative ways of thinking about physical and engineering problems and also is particularly rich in terms of available scientific literature and many

outstanding textbooks. Almost every new concept encountered can be thought of in different ways, and the interested reader can explore its implications further. We encourage such scholarly pursuit of enhanced knowledge and understanding by providing many footnotes in each chapter that provide further comments, qualifications of statements made in the text, and references for in-depth analyses of selected concepts and applications. Over 550 footnotes are spread over eleven chapters. These footnotes do not interrupt the flow of ideas and the development of the main topics, but they provide an unusual degree of completeness for a textbook at this level, with interesting and sometimes thought-provoking content to make the subject more appealing and satisfying.

## Electromagnetics and Waves in Engineering

The particular organization of this textbook, as well as its experimentally and physically based philosophy, are motivated by our view of the current status of electromagnetics in engineering curricula. Understanding electromagnetics and appreciating its applications require a generally higher level of abstraction than most other topics encountered by electrical engineering students. Beginning electrical engineers learn to deal with voltages and currents, which appear across or flow through circuit elements or paths. The relationships between these voltages and currents are determined by the characteristics of the circuit elements and by Kirchhoff's current and voltage laws. Voltages and currents in lumped electrical circuits are scalar quantities that vary only as a function of time, and are readily measurable, and the students can relate to them via their previous experiences. The relationships between these quantities (i.e., Kirchhoff's laws) are relatively simple algebraic or ordinary differential equations. On the contrary, electric and magnetic fields are *three-dimensional* and *vector* quantities that in general *vary in both space and time* and are related to one another through relatively complicated vector *partial differential* or vector *integral equations*. Even if the physical nature of electric and magnetic fields were understood, visualization of the fields and their effects on one another and on matter requires a generally high level of abstract thinking.

Most students are exposed to electromagnetics first at the freshman physics level, where electricity and magnetism are discussed in terms of their experimental bases by citing physical laws (e.g., Coulomb's law) and applying them to relatively simple and symmetrical configurations where the field quantities behave as scalars, and the governing equations are reduced to either algebraic equations of first-order integral or differential relationships. Freshman physics provides the students with their first experiences with fields and waves as well as some of their measurable manifestations, such as electric and magnetic forces, electromagnetic induction (Faraday's law), and refraction of light by prisms.

The first course in electromagnetics, which most students take after having had vector calculus, aims at the development and understanding of Maxwell's equations, requiring the utilization of the full three-dimensional vector form of the fields and their relationships. It is this very step that makes the subject of electromagnetics appear insurmountable to many students and turns off their interest, especially when coupled with a lack of presentation and discussion of important applications and the physical (and

experimental) bases of the fundamental laws of physics. Many authors and teachers have attempted to overcome this difficulty by a variety of topical organizations, ranging from those that start with Maxwell's equations as axioms to those that first develop them from their experimental basis.

Since electromagnetics is a mature basic science, and the topics covered in introductory texts are well established, the various texts primarily differ in their organization as well as range and depth of coverage. Teaching electromagnetics was the subject of a special issue of *IEEE Transactions on Education* [vol. 33, February, 1990]. Many of the challenges and opportunities that lie ahead in this connection were summarized well in an invited article by J. R. Whinnery.<sup>1</sup> Challenges include (1) the need to return to fundamentals (rather than relying on derived concepts), especially in view of the many emerging new applications that exploit unusual properties of materials and that rely on unconventional device concepts,<sup>2</sup> submillimeter transmission lines,<sup>3</sup> and optoelectronic waveguides,<sup>4</sup> and (2) the need to maintain student interest in spite of the decreasing popularity of the subject of electromagnetics and its reputation as a difficult and abstract subject.<sup>5</sup> Opportunities are abundant, especially as engineers working in the electronics industry discover that as devices get smaller and faster, circuit theory is insufficient in describing system performance or facilitating design. Transmission line concepts are not only important in microwave and millimeter-wave applications; due to modern GHz clock rates and nano-scale fabrication technology, they are also necessary in high-speed digital electronics, microelectronics, integrated circuits, interconnects,<sup>6</sup> and packaging applications.<sup>7</sup> In addition, issues of electromagnetic interference (EMI) and electromagnetic compatibility (EMC) limit the performance of system-, board-, and chip-level designs, and electrostatic discharge phenomena have significant impacts on the design and performance of integrated circuits.<sup>8</sup> The need for a basic understanding of electromagnetic waves and their guided propagation is underscored by the explosive expansion of the use of optical fibers, which enables extremely high data rates, ranging to 100 Gbits/s.<sup>9</sup> Fundamental bandwidth and power constraints in traditional copper-based transmission lines is also driving the development of optical interconnects with per-channel bandwidths

<sup>1</sup>J. R. Whinnery, The teaching of electromagnetics, *IEEE Trans. on Education*, 33(1), pp. 3–7, February 1990.

<sup>2</sup>D. Goldhaber-Gordon, M. S. Montemerlo, J. C. Love, G. J. Opitck, and J. C. Ellenbogen, Overview of nanoelectronic devices, *Proc. IEEE*, 85(4), pp. 521–540, April 1997.

<sup>3</sup>L. P. B. Katehi, Novel transmission lines for the submillimeter region, *Proc. IEEE*, 80(11), pp. 1771–1787, November 1992.

<sup>4</sup>R. A. Soref, Silicon-based optoelectronics, *Proc. IEEE*, 81(12), December, 1993.

<sup>5</sup>M. N. O. Sadiku, Problems faced by undergraduates studying electromagnetics, *IEEE Trans. Education*, 29(1), pp. 31–32, February, 1986.

<sup>6</sup>S. H. Hall and L. H. Heck, *Advanced signal integrity for high-speed digital designs*, John Wiley & Sons, 2011.

<sup>7</sup>H. B. Bakoglu, *Circuits, Interconnections, and Packaging for VLSI*, Addison Wesley, 1990.

<sup>8</sup>J. E. Vinson and J. J. Liou, Electrostatic discharge in semiconductor devices: an overview, *Proc. IEEE*, 86(2), pp. 399–418, February 1998.

<sup>9</sup>N. Cvijetic, D. Qian, and J. Hu, 100 Gb/s optical access based on optical orthogonal frequency-division multiplexing, *Communications Magazine, IEEE* 48(7), pp. 70–77, 2010.

in excess of 10 Gbits/s for high-performance computing applications.<sup>10</sup> Other important applications that require better understanding of electromagnetic fields are emerging in biology<sup>11</sup> and medicine.<sup>12</sup>

In organizing the material for our text, we benefited greatly from a review of the electromagnetic curriculum at Stanford University that one of us conducted during the spring quarter of 1990. A detailed analysis was made of both undergraduate and graduate offerings, both at Stanford and selected other schools. Inquiries were also made with selected industry, especially in the aerospace sector. Based on the responses we received from many of our colleagues, and based on our experience with the teaching of the two-quarter sequence at Stanford, it was decided that an emphasis on fundamentals and physical insight and a traditional order of topics would be most appropriate. It was also determined that transmission line theory and applications can naturally be studied before fields and waves, so as to provide a smooth transition from the previous circuits and systems experiences of the typical electrical engineering students and also to emphasize the importance of these concepts in high-speed electronics and computer applications.

### New to this Edition

This book represents an effort to merge the most important concepts from our two previous textbooks: *Engineering Electromagnetics*<sup>13</sup> and *Electromagnetic Waves*<sup>14</sup>. Some of the advanced topics from these two books, such as using transmission lines as resonant circuits and cylindrical waveguides, were moved to a web addendum (see Online Addendum section below). By moving some of these sections to the web, we are better able to focus the reader on the core concepts central to transmission lines, electromagnetics, and electromagnetic waves. We also introduce two new sections on increasingly relevant modern topics: Microelectromechanical Systems (MEMS) and Metamaterials. While these are relatively advanced topics, some of the fundamental physics underpinning these two areas of active research and development connect directly to the core ideas presented in this book, and so they give concrete examples of how a solid foundation in electromagnetics and waves is still very relevant to modern technology.

The list below summarizes the changes and additions we introduced in the second edition:

- We merged topics from the two first edition textbooks into a single volume covering both engineering electromagnetics and electromagnetic waves.
- We added two new sections: Microelectromechanical Systems (Section 4.14) and Metamaterials (Section 11.3).

<sup>10</sup>L. Chrostowski and K. Iniewski (Eds.), *High-speed Photonics Interconnects (Vol. 13)*, CRC Press., 2013.

<sup>11</sup>R. H., Funk, T. Monsees, and N. Ozkucur, Electromagnetic effects—From cell biology to medicine. *Progress in histochemistry and cytochemistry*, 43(4), 177-264, 2009.

<sup>12</sup>E. J. Bond, et al., Microwave imaging via space-time beamforming for early detection of breast cancer, *Antennas and Propagation, IEEE Transactions on*, 51(8), pp. 1690-1705, 2003.

<sup>13</sup>U.S. Inan and A. S. Inan, *Engineering Electromagnetics*, Addison Wesley Longman, 1999.

<sup>14</sup>U.S. Inan and A. S. Inan, *Electromagnetic Waves*, Prentice Hall, 2000.

- We added an appendix with a proof of the uniqueness theorem for Poisson’s equation (Appendix B).
- We moved several advanced topics in the first two editions to standalone addendum chapters that are available on the web.
- We introduced numerous edits throughout the text to add clarity and improve the presentation of some of the more challenging topics.
- We updated, modified, or added over 100 new end-of-chapter problems.
- We corrected errata that have been reported since the publication of the first two editions.

## Recommended Course Content

The wider breadth of topics covered by this single volume allows the instructor to tailor the content based on the duration of the course. Tables 1 and 2 list the suggested course content for a single course and two course sequence, respectively. Each table details suggested content based on the quarter system (32 contact hours per course per quarter) and semester system (42 contact hours per course per semester). The sections marked under “Cover” are recommended for complete coverage, including illustrative examples, whereas those marked “Skim” are recommended to be covered lightly, although the material provided is more complete in case individual students want to have more in-depth coverage. The one-course sequences provide the students with (1) a working knowledge of transmission lines, (2) a solid, physically based background and a firm understanding of Maxwell’s equations and their experimental bases, and (3) an introduction to electromagnetic waves. In addition to a more in-depth coverage of the transmission lines chapters and the development of Maxwell’s equations, the two-course sequences give the student a working knowledge of electromagnetic wave phenomena and their applications.

**TABLE 1** SUGGESTED COURSE CONTENT: SINGLE QUARTER OR SEMESTER

Chapter	Quarter Course (32 Hours)		Semester Course (42 Hours)	
	Cover	Skim	Cover	Skim
1	All		All	
2	2.1–2.4	2.7	2.1 – 2.5	2.7
3	3.1–3.3		3.1–3.6	
4	4.1–4.9	4.10	4.1–4.10	4.12
5	5.1–5.5		5.1–5.5	5.7
6	6.1–6.7	6.8	6.1–6.8	6.10
7	7.1, 7.2, 7.4	7.3, 7.5	7.1, 7.2, 7.4	7.3, 7.5
8	8.1, 8.2		8.1–8.4	
9				
10				
11				

**TABLE 2** SUGGESTED COURSE CONTENT: TWO QUARTERS OR SEMESTERS

Chapter	Two Quarter Course (64 Hours Total)		Two Semester Courses (84 Hours Total)	
	Cover	Skim	Cover	Skim
1	All		All	
2	All		All	
3	3.1–3.6		All	
4	4.1–4.12		4.1–4.13	4.14
5	5.1–5.7	5.8	All	
6	6.1–6.9		All	
Quarter Break			Semester Break	
7	All		All	
8	8.1–8.6		All	
9	9.1–9.3, 9.5–9.7		All	
10	All		All	
11	11.1, 11.2		11.1, 11.2,	11.3

### Instructor's Manual

We firmly believe that practice is the key to learning and that homework and exams are all instruments of teaching—although they may not be regarded as such by the students at the time. In our own courses, we take pride in providing the students with detailed solutions of homework and exam problems, rather than cryptic and abbreviated answers. To aid the instructors who choose to use this text, we have thus taken it upon ourselves to prepare a well-laid-out solutions manual, describing the solution of *every* end-of-chapter problem, in the same step-by-step detailed manner as our illustrative examples within the chapters. This instructor's manual is available to instructors upon request at [www.pearsonhighered.com](http://www.pearsonhighered.com). Supplemental information about the book and errata will be available at [www.pearsonhighered.com/inan](http://www.pearsonhighered.com/inan).

As authors of this book, we are looking forward to interacting with its users, both students and instructors, to collect and respond to their comments, questions, and corrections. We can most easily be reached by electronic mail at [inan@stanford.edu](mailto:inan@stanford.edu) (<http://vlf.stanford.edu/>), [uinan@ku.edu.tr](mailto:uinan@ku.edu.tr) (<http://www.ku.edu.tr/en/about-ku/president>), [ainan@up.edu](mailto:ainan@up.edu) ([faculty.up.edu/ainan/](http://faculty.up.edu/ainan/)), and [ryan.said@vaisala.com](mailto:ryan.said@vaisala.com).

### Online Addendum

The topics in this book were carefully selected to give the student a solid foundation of transmission lines, Maxwell's equations, and the propagation and guiding of electromagnetic waves. The goal of this book is to develop an intuitive understanding of these fundamental concepts, so that the student is well equipped to apply these principles to

**TABLE 3** ADDENDUM: ADVANCED TOPICS (AVAILABLE ONLINE)

Addendum A	Transmission Lines: Advanced Topics
Addendum B	Miscellaneous Wave Topics
Addendum C	Cylindrical Waveguides
Addendum D	Cavity Resonators
Addendum E	Field-Matter Interactions: Advanced Topics
Addendum F	Electromagnetic Radiation and Elementary Antennas

new challenges and to expand his/her study to more advanced topics. Due to the breadth of discussion given to each topic, in order to maintain a manageable page count, a few of the more advanced topics from the first edition are moved to an online addendum. This addendum, which is available for free at [www.pearsonhighered.com/inan](http://www.pearsonhighered.com/inan), contains the chapters listed in Table 3.

The Addendum chapters cover advanced topics that expand on the core material of this text. Addendum A includes selected advanced transmission line topics, including transients on lossy transmission lines and the use of transmission lines as resonant circuits. The application examples of Addendum A present further reading that can be covered after Chapters 2 and 3. The remaining addendum chapters cover advanced material related to the subject matter from the final four chapters of the book. Addendum B expands on two advanced topics introduced in Chapters 8 and 9: examples of nonplanar waves and oblique-incidence reflection from a good conductor. Addendum C extends the treatment of planar waveguides from Chapter 10 to those that are bounded in two dimensions. Addendum D introduces cavity resonators, whose treatment follows naturally from the two-dimensional waveguides covered in Addendum C. Addendum E extends the field-matter interaction topics encountered in Sections 11.1 and 11.2. Addendum F introduces electromagnetic radiation and elementary antennas, which connect the electromagnetic fields studied in Chapters 8–11 to their sources.

## Acknowledgments

We gratefully acknowledge those who have made significant contributions to the successful completion of this text. We thank Professor J. W. Goodman of Stanford, for his generous support of textbook writing by faculty throughout his term as department chair. We thank Professor Gordon Kino and Dr. Timothy F. Bell of Stanford, for course-testing a preliminary version of the manuscript for our first edition. We thank numerous colleagues and former students who have identified errors and suggested clarifications. We thank Mrs. Jun-Hua Wang for typing parts of the first edition manuscript and drawing some of the illustrations. We thank Dr. Robert Marshall of Stanford for preparing the two first edition manuscripts for use in a single unified book, and Güneş Aydınođan for typesetting large portions of the solution manual. We owe special thanks to our reviewers on the first edition for their valuable comments and suggestions, including J. Bredow of University of Texas—Arlington; S. Castillo of New Mexico State University; R. J. Coleman of University of North Carolina—Charlotte; A. Dienes of University of California—Davis; J. Dunn

of University of Colorado; D. S. Elliott of Purdue University; R. A. Kinney of Louisiana State University; L. Rosenthal of Fairleigh Dickinson University; E. Schamiloglu of University of New Mexico; T. Shumpert of Auburn University; D. Stephenson of Iowa State University; E. Thomson of University of Florida; J. Volakis of University of Michigan; and A. Weisshaar of Oregon State University.

We greatly appreciate the efforts of our managing editor at Pearson, Scott Disanno, and his staff, including Michelle Bayman, William Opaluch, Joanne Manning, and Julie Bai. We also thank Haseen Khan and her staff at Laserwords for their dedication and attention to detail in the layout and production of the book.

We dedicate this book to our parents, Mustafa and Hayriye Inan, for their dedication to our education; to our wives, Elif and Belgin, for their persistent support and understanding as this project expanded well beyond our initial expectations and consumed most of our available time for too many years; and to our children, Ayse, Ali, Baris, and Cem, and grandchildren, Ayla and Nisa, for the joy they bring to our lives.

*Umran S. Inan*

*Aziz S. Inan*

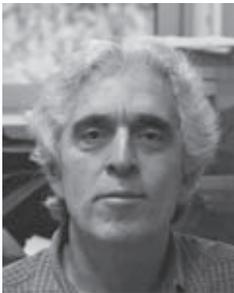
I would like to thank Professors Umran Inan and Aziz Inan for giving me the opportunity to contribute to this manuscript. When I was a new graduate student at Stanford University, I had the pleasure of learning electromagnetics from the two first edition textbooks that form the basis for this second edition. It is an honor to help create a unified book that spans the content of these courses that I so enjoyed taking over a decade ago. I dedicate this book to my parents, James and Connie, for cultivating my passion for science, and to my sister, Amirah, for her unwavering moral support.

*Ryan K. Said*

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# Introduction

This book is an introduction to the fundamental principles and applications of *electromagnetics*. The subject of electromagnetics encompasses *electricity*, *magnetism*, and *electrodynamics*, including *all* electric and magnetic phenomena and their practical applications. A branch of electromagnetics, dealing with electric charges at rest (static electricity) named *electrostatics*, provides a framework within which we can understand the simple fact that a piece of amber, when rubbed, attracts itself to other small objects.<sup>1</sup> Another branch dealing with static magnetism, namely *magnetostatics*, is based on the facts that some mineral ores (e.g., lodestone) attract iron<sup>2</sup> and that current-carrying wires produce magnetic fields.<sup>3</sup> The branch of electromagnetics known as *electrodynamics* deals with the time variations of electricity and magnetism and is based on the fact that magnetic fields that change with time produce electric fields.<sup>4</sup>

Electromagnetic phenomena are governed by a compact set of principles known as Maxwell's equations,<sup>5</sup> the most fundamental consequence of which is that electromagnetic energy can propagate, or travel from one point to another, as *waves*. The propagation of electromagnetic waves results in the phenomenon of *delayed action at a distance*; in other words, electromagnetic fields can exert forces, and hence can do work, at distances far away from the places where they are generated and at later times. Electromagnetic radiation is thus a means of transporting energy and momentum from one set of electric charges and currents (at the source end) to another (those at the receiving end).

<sup>1</sup>First discovered by the Greek mathematician, astronomer, and philosopher Thales of Miletus [640–548 B.C.].

<sup>2</sup>First noted by the Roman poet and philosopher Lucretius [99?–55? B.C.], in his philosophical and scientific poem titled *De rerum natura (On the Nature of Things)*.

<sup>3</sup>First noted by Danish physicist H. C. Oersted in 1819.

<sup>4</sup>First noted by British scientist M. Faraday in 1831.

<sup>5</sup>J. C. Maxwell, *A Treatise in Electricity and Magnetism*, Clarendon Press, Oxford, 1892, Vol. 2, pp. 247–262.

Since whatever can carry energy can also convey information, *electromagnetic waves* thus provide the means of transmitting energy and information at a distance. The latter half of this book provides an introduction to electromagnetic waves, their propagation in empty space or material media, their reflection from boundaries, and their guiding within planar boundaries.

The concept of waves is one of the great unifying ideas of physics and engineering.<sup>6</sup> Our physical environment is full of waves of all kinds: seismic waves, waves on oceans and ponds, sound waves, heat waves, and even traffic waves. The idea of delayed action as manifested in wave phenomena is familiar to us when we hear a sound and its echo or when we create a disturbance<sup>7</sup> in a pool of water and observe that waves reach the edge of the pool after a noticeable time. We also appreciate that it might take minutes or hours for heat to penetrate into objects; that the thunderclap is delayed with respect to the lightning flash by many seconds; and that when we are lined up in front of a traffic light, it often takes a long time for us to be able to move after the light turns green. Light, or electromagnetic waves, travel so fast that their delayed action is not perceptible to our senses in our everyday experiences. On the other hand, in astronomy and astrophysics we deal with vast distances; light waves from a supernova explosion may arrive at earth millions of years after the brightness that created them has been extinguished.

**Example 1.1: Time delay between Mars and Earth.** The distance between Earth and Mars varies from  $54.6 \times 10^6$  to  $401 \times 10^6$  km. How long does it take for a message sent from Earth to reach NASA's Curiosity Rover on Mars?

**Solution:** The travel time  $t$  is determined by the distance  $l$  and speed of propagation  $v$ :

$$t = \frac{l}{v}$$

All electromagnetic waves, including radio waves used in broadcast telecommunications, travel in free space at the speed of  $c \simeq 3 \times 10^8$  m-s<sup>-1</sup>. Using  $v = c$ , the travel time from Earth to Mars ranges from

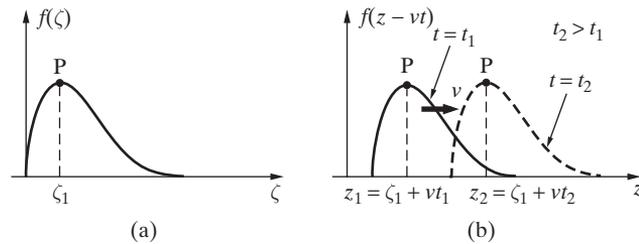
$$t_{\min} = \frac{54.6 \times 10^6 \text{ km}}{3 \times 10^5 \text{ km-s}^{-1}} \cdot \frac{1 \text{ minute}}{60 \text{ s}} \simeq 3.03 \text{ minutes}$$

to

$$t_{\max} = \frac{401 \times 10^6 \text{ km}}{3 \times 10^5 \text{ km-s}^{-1}} \cdot \frac{1 \text{ minute}}{60 \text{ s}} \simeq 22.3 \text{ minutes}$$

<sup>6</sup>For an excellent qualitative discussion, see J. R. Pierce, *Almost All About Waves*, MIT Press, Cambridge, Massachusetts, 1974. For more extensive treatment of waves of all kinds, see K. U. Ingard, *Fundamentals of Waves and Oscillations*, Cambridge University Press, Cambridge, England, 1990.

<sup>7</sup>On the scale of a pond, we can simply think of dropping a stone; on a larger scale, earthquakes in oceans produce giant *tsunami* waves. A 9-meter-high tsunami produced by the 1964 Alaskan earthquake hit the Hawaiian islands (at a distance of 2000 km) about 5 hours later, causing more than 25 million dollars of damage.



**Figure 1.1** Example of a wave. (a) An arbitrary function  $f(\zeta)$ . (b) The function  $f(z - vt)$ , where  $v$  is a positive constant, plotted versus  $z$  at  $t = t_1$  and  $t = t_2$ . The wave nature is evident as the pattern in space at time  $t_1$  is shifted to other values of  $z$  at a later time  $t_2$ .

At a qualitative level, we recognize a wave as some pattern in space that appears to move in time. Wave motion does not necessarily involve repetitive undulations of a physical quantity (e.g., the height of the water surface for water waves in a lake). If a disturbance that occurs at a particular point in space at a particular time is related in a definite manner to what occurs at distant points in later times, then there is said to be wave motion. To express this mathematically, let  $z$  be distance,  $t$  be time, and  $v$  be a fixed positive parameter. Consider any arbitrary function  $f(\zeta)$  of the argument  $\zeta = (z - vt)$ . Figure 1.1a shows a sketch of  $f(\zeta)$ , identifying a point P on the curve that corresponds to  $f(\zeta_1)$ . Note that this peak value of the function (i.e., point P) is obtained whenever its argument equals  $\zeta_1$ . Shown in Figure 1.1b is a sketch of  $f(z - vt_1)$  with respect to  $z$  at a fixed time  $t_1$ , where the point P is at  $z_1 = \zeta_1 + vt_1$ . A plot of the function at  $t = t_2 > t_1$  (i.e.,  $f(z - vt_2)$ ) is also shown in Figure 1.1b, where we see that the point P has now moved to the right, to the new location  $z_2 = \zeta_1 + vt_2$ . It is clear that the entire curve  $f(\zeta)$ , which comprises the function  $f(z - vt)$ , moves in the  $z$  direction as time elapses. The velocity of this motion can be determined by observing a fixed point on the curve—for example, point P. Since this point is defined by the argument of the function being equal to  $\zeta_1$ , we can set  $(z - vt) = \zeta_1$ , which upon differentiation yields  $dz/dt = v$ , since  $\zeta_1$  is a constant. It thus appears that the speed with which point P moves to the right is  $v$ , which is identified as the velocity of the wave motion. Note that the function  $f(\cdot)$  could represent any physically observable entity; it may be a scalar,<sup>8</sup> such as voltage, or it may be a vector, such as the velocity of an object in motion. If  $f(\cdot)$  is a vector, each of its components must be a function of  $(z - vt)$  for it to be a propagating wave. Quantities varying as functions of  $(z - vt)$  constitute natural solutions of the fundamental equations of electromagnetics and distributed electrical circuits. Chapters 2, 3, and 8–11 of this textbook are devoted to the study of voltage, current, and electromagnetic waves that vary in space and time as functions of  $(z - vt)$ .

<sup>8</sup>A scalar is a quantity that is completely specified by its value, such as the number of coins in your pocket, the number of people or the density of air in a room, pressure, or temperature. Other physical quantities have direction; for example, velocity, momentum, force, or displacement. Specification of a vector quantity requires both a magnitude and direction. A brief review of basic principles of vector analysis is provided in Appendix A.

Most waves travel through substances, whether they be earth, water, air, steel, or quartz, without actually carrying the substance bodily with them.<sup>9</sup> Like moving objects, traveling waves carry energy, albeit by different amounts depending on the nature of the waves and the medium they propagate in. Electromagnetic waves have the special property that they can also propagate in vacuum, without any matter present. However, the propagation of electromagnetic waves is nevertheless affected by the presence of matter, and this property often allows us to confine or guide waves and in doing so utilize them more efficiently. Electromagnetic engineering problems generally involve the design and use of materials that can generate, transmit, guide, store, distribute, scatter, absorb, and detect electromagnetic waves and energy.

The 20th century has witnessed rapid advances in electrical engineering, which have largely come about by our ability to predict the performance of sophisticated electrical circuits accurately. Central to this tremendous progress is our ability to utilize the simple but powerful tool called electric *circuit theory*. Classical circuit theory considers a voltage or current source applied to an electrical circuit consisting of series and/or parallel connection of simple *lumped* (see Section 1.1) circuit elements, such as resistances, capacitances, inductances and dependent sources, which may be idealized models of more complex physical devices. The behavior of circuits is described by ordinary differential equations that are derived on the basis of Kirchhoff's voltage and current laws. Circuit theory is a simplified approximation to the more exact electromagnetic theory.<sup>10</sup> The classical theory of electricity and magnetism relies on a set of physical laws known as Maxwell's equations, which are based on experimental facts and which govern all electromagnetic phenomena. Electromagnetic theory is inherently more complicated than circuit theory, primarily because of the larger number of variables involved. In general electromagnetic problems, most of the physical quantities that we deal with are vectors, whose values may depend on all three coordinates of space (i.e.,  $x$ ,  $y$ , and  $z$  in rectangular coordinates) and time ( $t$ ). In classical circuit theory, on the other hand, voltages and currents are scalar quantities and are typically functions of only one variable, namely time. The theory of *distributed* circuits (see Section 1.1), or transmission lines, represents an intermediate level of complexity where, in many cases, we can continue to deal with scalar quantities, such as voltages and currents, that are now functions of two variables, namely a single spatial dimension and time. In this regime, we can continue to benefit from the relative simplicity of circuit theory, while treating problems for which the lumped circuit theory is not applicable.

In this text, and in view of the preceding discussion, we choose to study distributed circuits or transmission lines using a natural extension of circuit theory before we formally introduce the physical laws of electricity and magnetism. This approach presents the general fundamental concepts of waves and oscillations at the outset, which

<sup>9</sup>Leonardo da Vinci [1452–1519] wrote of waves, “The impetus is much quicker than the water, for it often happens that the wave flees the place of its creation, while the water does not; like the waves made in a field of grain by the wind, where we see the waves running across the field while the grains remain in place” [J. R. Pierce, *Almost All About Waves*, MIT Press, Cambridge, Massachusetts, 1974].

<sup>10</sup>Kirchhoff's voltage and current laws, which provide the basis for classical circuit theory, can be derived from the more general electromagnetics equations; see Chapter 4 of S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*, 3rd ed., John Wiley & Sons, Inc., New York, 1994.

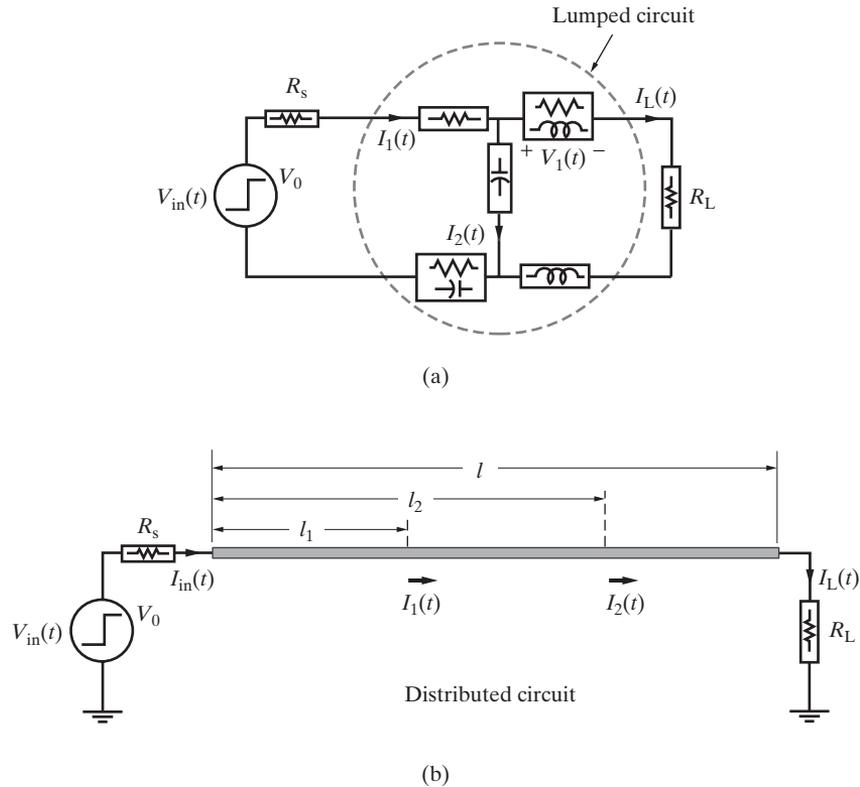
are expanded upon later when we study propagation of electromagnetic waves. In this way, the reader is provided an unhindered initial exposure to properties of waves such as frequency, phase velocity, wavelength, and characteristic impedance; to energy relations in oscillating systems and waves; and to concepts such as reflection and bandwidth, as well as to fundamental mathematical techniques necessary to describe waves and oscillations. All of these concepts subsequently extend to more complicated problems and applications where a full electromagnetic treatment becomes necessary. An initial exposure to transmission line analysis also enables us to address a wide range of increasingly important engineering applications that require the use of wave techniques, but for which a full vector electromagnetic analysis may not be necessary.<sup>11</sup> In our coverage of transmission lines or distributed circuits, we assume that the reader is familiar with the elementary physics of electricity and magnetism (at the level of freshman physics) and with electrical circuits at the level of understanding Kirchhoff's voltage and current laws and terminal behavior (i.e., voltage-current relationships) of circuit elements such as inductors, capacitors, and resistors. A more complete discussion of the concepts of inductance, capacitance, and resistance is provided in later chapters using the concepts and principles of electromagnetic fields as we introduce the fundamental laws of electromagnetics.

## 1.1 LUMPED VERSUS DISTRIBUTED ELECTRICAL CIRCUITS

A typical electrical engineering student is familiar with circuits, which are described as *lumped*, *linear*, and *time-invariant* systems and which can be modeled by *ordinary*, *linear*, and *constant-coefficient* differential equations. The concepts of linearity and time invariance refer to the relationships between the inputs and outputs of the system. The concept of a lumped circuit refers to the assumption that the entire circuit (or system) is at a single point (or in one "lump"), so that the dimensions of the system components (e.g., individual resistors or capacitors) are negligible. In other words, current and voltage do not vary with space across or between circuit elements, so that when a voltage or current is applied at one point in the circuit, currents and voltages of all other points in the circuit react instantaneously. Lumped circuits consist of interconnections of lumped elements. A circuit element is said to be lumped if the instantaneous current entering one of its terminals is equal to the instantaneous current throughout the element and leaving the other terminal. Typical lumped circuit elements are resistors, capacitors, and inductors. In a lumped circuit, the individual lumped circuit elements are connected to each other and to sources and loads within or outside the circuit by conducting paths of negligible electrical length.<sup>12</sup> Figure 1.2a illustrates a lumped electrical circuit to which an input voltage of  $V_{\text{in}} = V_0$  is applied at  $t = 0$ . Since the entire circuit is considered in one lump, the effect of the input excitation is instantaneously felt at all points in the

<sup>11</sup>Examples are on-chip and chip-to-chip interconnections in digital integrated circuits and many other computer engineering applications. See A. Deutsch, et al., When are transmission-line effects important for on-chip interconnections, *IEEE Trans. Mic. Th. MTT*, 45(10), pp. 1836–1846, October 1997.

<sup>12</sup>The electrical length equals the physical length of the circuit element divided by the wavelength. As we discuss in Section 1.1.3, this ratio determines whether a given circuit element should be treated as a lumped or distributed element.



**Figure 1.2 Lumped versus distributed electrical circuits.** (a) When a step voltage  $V_{in}(t)$  is applied to a lumped circuit, we assume that all currents and voltages start to change at  $t = 0$ , implicitly assuming that it takes zero travel time for the effect of the input to move from any point to any other point in the circuit. (b) In a distributed circuit, the nonzero travel time of the signal from one point to another cannot be neglected. For example, when a step voltage  $V_{in}(t)$  is applied at one end of the circuit at  $t = 0$ , the load current at the other end does not start to change until  $t = l/v$ .

circuit, and all currents and voltages (such as  $I_1$ ,  $I_2$ ,  $V_1$ , and the load current  $I_L$ ) either attain new values or respond by starting to change at  $t = 0$ , in accordance with the natural response of the circuit to a step excitation as determined by the solution of its corresponding differential equation. Many powerful techniques of analysis, design, and computer-aided optimization of lumped circuits are available and widely used.

The behavior of lumped circuits is analogous to rigid-body dynamics. In mechanics, a rigid body is postulated to have a definite shape and mass, and it is assumed that the distance between any two points on the body does not change, so that its shape is not deformed by applied forces. Thus, an external force applied to a rigid body is assumed to be felt by all parts of the body simultaneously, without accounting for the finite time it would take for the effect of the force to travel elastically from one end of the body to another.

With the “lumped” assumption, one does not have to consider the travel time of the signal from one point to another. In reality, however, disturbances or signals caused by any applied energy travel from one point to another in a nonzero time. For electromagnetic signals, this travel time is determined by the speed of light,<sup>13</sup>  $c \simeq 3 \times 10^8 \text{ m}\cdot\text{s}^{-1} = 30 \text{ cm}\cdot(\text{ns})^{-1}$ . In practical transmission systems, the speed of signal propagation is determined by the electrical and magnetic properties of the surrounding media and the geometrical configuration of the conductors and may in general be different from  $c$ , but it is nevertheless of the same order of magnitude as  $c$ . Circuits for which this nonzero travel time cannot be neglected are known as *distributed* circuits. An example of a distributed circuit is a long wire, as shown in Figure 1.2b. When an input voltage  $V_0$  is applied at the input terminals of such a distributed circuit (i.e., between the input end of the wire and the electrical ground) at  $t = 0$ , the voltages and currents at all points of the wire cannot respond simultaneously to the applied excitation because the energy corresponding to the applied voltage propagates down the wire with a finite velocity  $v$ . Thus, while the input current  $I_{\text{in}}(t)$  may change from zero to  $I_0$  at  $t = 0$ , the current  $I_1(t)$  does not flow until after  $t = l_1/v$ ,  $I_2(t)$  does not flow until after  $t = l_2/v$ , and no load current  $I_L(t)$  can flow until after  $t = l/v$ . Similarly, when a harmonically (i.e., sinusoidally) time-varying voltage is connected to such a line, the successive rises and falls of the source voltage propagate along the line with a finite velocity so that the currents and voltages at other points on the line do not reach their maxima and minima at the same time as the input voltage.

In view of the fundamentally different behavior of lumped and distributed circuits as illustrated in Figure 1.2, it is important, in practice, to determine correctly whether a lumped treatment is sufficiently accurate or whether the circuit in hand has to be treated as a distributed circuit. In the following, we quantify on a heuristic basis the circumstances under which the travel time and/or the physical size of the circuit components or the length of the interconnects between them can be neglected. Note that in problems related to heat, diffusion, sound waves, water waves, traffic waves, and so on, the travel time is readily observable and almost always has to be accounted for. In the context of electromagnetics, on the other hand, we find a wide range of applications where lumped analysis is sufficiently accurate, is substantially simpler, and provides entirely satisfactory results. However, in an equally wide range of other applications we find that the lumped treatment is not sufficiently accurate and that one has to resort to field and wave techniques, which are generally more involved, both mathematically and conceptually. It is thus important, particularly in the context of electromagnetic applications, that we develop criteria by which we can determine the applicability of lumped circuit formulations. We provide below a heuristic discussion from three different but related points of view.

### 1.1.1 Rise Time versus Travel Time

It is apparent from the preceding discussion that we can consider the delay time (travel time) over the signal path as long or short, important or negligible only relative to

<sup>13</sup>The more accurate empirical value of the speed of light is  $c = 299,792,458 \text{ m}\cdot\text{s}^{-1}$  [CRC Handbook of Chemistry and Physics, 76th ed., CRC Press, Inc., Boca Raton, Florida, 1995].

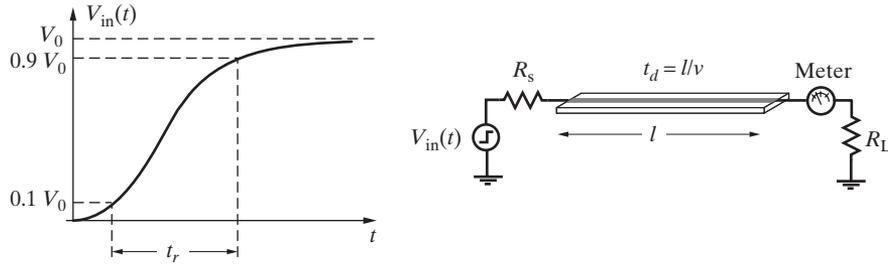


Figure 1.3 Rise time versus one-way travel time.

or in comparison with some other quantity. In terms of positive step changes in an applied signal, we note that the input signal typically exhibits a nonzero *rise time* (usually measured as the time required for the signal to change from 10% to 90% of its final value), which may be denoted as  $t_r$  (Figure 1.3). We can then compare  $t_r$  to the one-way propagation time delay through the signal path (also called the one-way transit time or time of flight),  $t_d = l/v$ , where  $v$  is the velocity of propagation and  $l$  is the length of the signal path. For example, in practical design of interconnects between integrated circuit chips, one rule of thumb is that the signal path can be treated as a lumped element if  $(t_r/t_d) > 6$ , whereas lumped analysis is not appropriate for  $(t_r/t_d) < 2.5$ . Whether lumped analysis is appropriate for the in-between range of  $2.5 < (t_r/t_d) < 6$  depends on the application in hand and the required accuracy. To summarize:

$$\begin{array}{ll} (t_r/t_d) > 6 & \text{(lumped)} \\ (t_r/t_d) < 2.5 & \text{(distributed)} \end{array} \quad (1.1)$$

In terms of the particular application of high-speed integrated circuits, the on-chip rise times range from 0.5–2 ns for CMOS to 0.02–0.1 ns for GaAs technologies. The speed of signal propagation within the chips depends on the material properties; for example, it is  $\sim 0.51c$  for  $\text{SiO}_2$ . Thus, for on-chip interconnections (typical  $l \simeq 1$  cm), lumped circuit analysis breaks down (i.e.,  $t_r/t_d < 2.5$ ) for rise times less than  $\sim 0.165$  ns. For printed circuit boards made of a commonly used glass epoxy material, the speed of propagation is  $\sim 0.47c$ , so that for a  $\sim 10$  cm interconnect, lumped analysis is not appropriate for rise times less than  $\sim 1.8$  ns. As clock speeds increase and rise times become accordingly shorter, distributed analyses will be required in a wider range of digital integrated circuit applications.

The importance of considering rise time versus travel time is underscored by advances in the generation of picosecond pulses.<sup>14</sup> For such extremely short pulse durations, with subpicosecond rise times, distributed circuit treatment becomes necessary for one-way propagation time delay of  $t_d > 10^{-13}$  seconds. Assuming propagation at the

<sup>14</sup>See D. W. Van der Weide, All-electronic generation of 0.88 picosecond, 3.5 V shockwaves and their application to a 3 Terahertz free-space signal generation system, *Appl. Phys. Lett.*, 62(1), pp. 22–24, January 1993.

speed of light, the corresponding distances are  $> 0.03$  mm! In other words, for picosecond rise times, lumped analysis is not appropriate for circuits with physical dimensions longer than a few tens of microns ( $1$  micron =  $1 \mu\text{m}$ ).

**Example 1.2: Lumped- or distributed-circuit element.** Consider a 10-cm long microstrip transmission line having a propagation speed of  $15 \text{ cm}\cdot\text{ns}^{-1}$ . For digital signal transmission with a 100 ps rise time, should this microstrip line be treated as a lumped- or distributed-circuit element?

**Solution:** The one-way time delay of the microstrip line can be found as

$$t_d = l/v = 10 \text{ cm}/15 \text{ cm}\cdot\text{ns}^{-1} \simeq 0.67 \text{ ns}$$

Based on rule-of-thumb criteria (1.1), since  $t_r/t_d \simeq 0.1 \text{ ns}/0.67 \text{ ns} = 0.15 < 2.5$ , the microstrip line should be modeled as a distributed-circuit element.

### 1.1.2 Period versus Travel Time

For sinusoidal steady-state applications, the suitability of a lumped treatment can be assessed by comparing the one-way propagation delay  $t_d$  with the period  $T$  of the propagating sine wave involved. As an illustration, consider the telephone line shown in Figure 1.4. If a sinusoidal voltage at frequency  $f$  is applied at the input of the line so that  $V_{AA'}(t) = V_0 \cos(2\pi ft)$ , the voltage at a distance of  $l$  from the input is delayed by the travel time,  $t_d = l/v$ . In other words,

$$V_{BB'}(t) = V_0 \cos[2\pi f(t - t_d)] = V_0 \cos\left[2\pi ft - 2\pi \frac{t_d}{T}\right] \quad (1.2)$$

where  $T = 1/f$  is the period of the sinusoidal signal. It is apparent from (1.2) that if  $t_d \ll T$ , then the voltage  $V_{BB'}(t)$  is very nearly the same as the input voltage  $V_{AA'}(t)$  at all times, and the telephone line can be treated as a lumped system. If, on the other

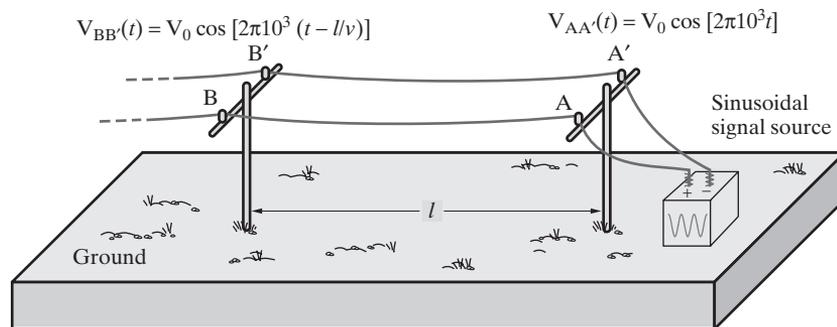


Figure 1.4 Period versus one-way travel time.